Fermi National Accelerator Laboratory

FERMILAB-Conf-98/001

Particle Diffusion in Overlapping Resonances

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January 1998

Published Proceedings of the *Advanced IFCA Workshop on Beam Dynamics Issues for e+e- Factories*, Frascati, Italy, October 20-25, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

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Distribution

Frascati Physics Series Vol. XXX (1997), pp. 000-000
Conference Title - Conference Town, Oct 3rd, 1996

PARTICLE DIFFUSION IN OVERLAPPING RESONANCES

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ABSTRACT

Longitudinal bunch dilution was studied in the IUCF Cooler Ring with phase modulation of a higher-order harmonic rf. Diffusion in the presence of overlapping parametric resonances has been identified as the dilution mechanism. We found that fast growth is associated with the rapid particle motion along separatrices of dominant parametric resonances, slow growth is related to particle diffusion in the chaotic sea, and saturation occurs when particles are bounded by an invariant torus.

1 Introduction

Longitudinal phase space dilution is important in the operation of synchrotrons, for example, in the mitigation of microwave instability and minimization of the negative mass instability when crossing transition. A common procedure is to modulate a secondary rf system. The process can be described by the Hamiltonian

$$H = \frac{1}{2}\nu_{s0}\delta^2 + \nu_{s0}\left\{[1-\cos h_1\phi] - \frac{r}{h_2}[1-\cos(h_2\phi+\Delta\phi)]\right\} \ , \qquad (1.1)$$

where h_1 and h_2 are the primary and secondary rf harmonics, r is the ratio of the two rf voltages, ν_{s0} is the synchrotron tune at zero amplitude when the secondary rf is absent, $\Delta\phi = \Delta\phi_0 + a\sin\nu_m\theta$ is the phase difference between the two rf systems with a the modulation strength and ν_m the modulation tune. Here, the normalized momentum spread has been chosen as $\delta = -(h_1|\eta|/\nu_{s0})(\Delta p/p_0)$, so that the small-amplitude trajectory is a circle if there is only one rf system. Kats [1)] studied the situation of $\Delta\phi_0 = 180°$. He converted the equation of motion into the Mathieu equation and identified the growth as driven by the 2:1 parametric resonance and the growth rate by the modulation amplitude a. Balandin, Dyachkov, and Shaposhnikova [2)] also identified the growth as driven by the $mk\!:\!m$ resonance, with $m = 1, 2, \cdots$. However, their derivation is valid only for a small bunch that does not grow by too much. Cappi, Garoby, and Shaposhnikova [3)] actually performed an experiment to verify the theoretical analysis, and found reasonably good agreement.

Here, we like to answer the questions: What is the dilution mechanism, especially when dilution is big? Is the dilution driven by one resonance or diffusion resulting from overlapping resonances? What provides the controlled growth?

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†Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

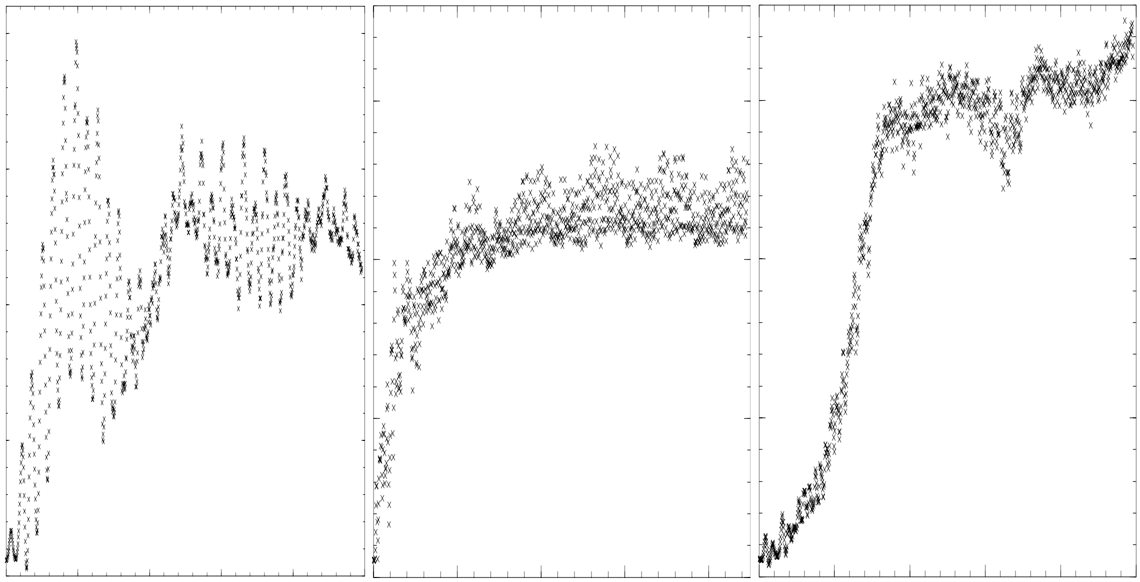

Figure 1: *Plots of σ_t^2 as functions of time for 3 situations; vertical scales arbitrary.*

2 THE EXPERIMENT

A study was performed on the IUCF Cooler Ring at Indiana, which has electron cooling. The study was done at 45 MeV with a revolution frequency of $f_0 = 1.03168$ MHz. Primary rf harmonic was $h_1 = 1$, secondary rf $h_2 = 9$. The electron-cooling time was roughly 0.4 s. Equilibrium bunch length was $\sigma_t = 12\pm1$ ns or $\sigma_\phi = 0.0778 \pm 0.0065$ rad. Small-amplitude synchrotron frequency with only the primary rf was $f_{s0} = \nu_{s0} f_0 \sim 667$ Hz.

An injected bunch was first cooled in the Cooler for 3 s with only the primary rf turned on. Then, the secondary rf cavity was turned on with a chosen relative phase $\Delta\phi_0$ and a chosen voltage ratio r (between the 2 rf's). At the same time, this relative phase $\Delta\phi_0$ was modulated with a frequency $f_m = \nu_m f_0$ and an amplitude a. The evolution of the beam profile was measured with a high-bandwidth BPM through a low-loss high-bandwidth cable. The beam profile was digitized by a digital scope with 512 channels, each having a resolution of 1 ns.

3 EXPERIMENTAL RESULTS

We plot σ_t^2 versus time t and expect $\sigma_t^2 \propto t$ when total chaos occurs according to the Einstein's relation, since diffusion is a random process. Typically, we see 4 types of σ_t^2 evolutions: little or no growth, growth with large amplitude oscillations, initial fast linear growth followed by linear growth with smaller slope and then saturation, initial nonlinear growth, which may be a combination of linear growths with larger and larger slopes. The last three situations are illustrated in Fig. 1.

For a perturbation analysis, we can rewrite the Hamiltonian in Eq. (1.1) as $H = H_0 + H_1$. Here, H_0 is in fact the Hamiltonian at zero amplitude modulation, and gives the correct phase-dependent tune of the *unperturbed* 2-rf system. If the modulation amplitude a is small, the tune vs phase plot will tell us about how

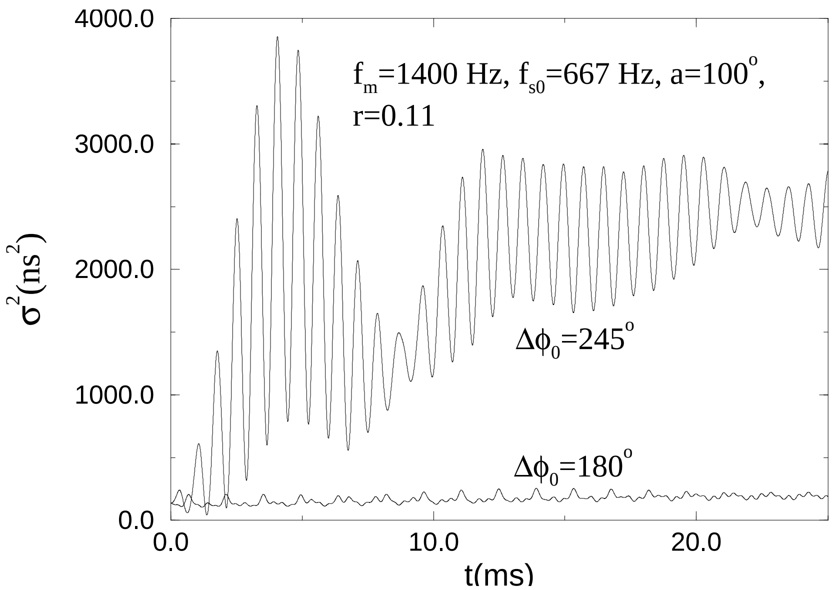

Figure 2: *Simulation results of σ_t^2 vs time for relative phase difference $\Delta\phi_0 = 180°$ and 245°. Other parameters are $f_m = 1400$ Hz, $a = 100°$, $r = 0.11$, and $h = 9$.*

parametric resonances come in. Unfortunately, the modulation amplitude a in this experiment is of the order of 100° or 1.75 rad. Higher-order perturbation will be important. Higher-order resonances will come into play leading to a chaotic region in the center of phase space. Therefore, we try to compare experimental results with simulations instead.

Damping and random excitation are introduced so that the initial rms bunch width is $\sigma_\phi \sim 0.078$ rad, when only the primary rf is present. The bunch particles are then tracked for 10^4 to 10^5 turns with the secondary rf and phase modulation turned on. There are 4 variables: modulation tune ν_m, about $2\nu_{s0}$ to $3\nu_{s0}$, modulation amplitude a, about 100°, ratio of rf voltages r, about 0.1 to 0.2, and relative phase between the 2 rf's $\Delta\phi_0$. For a rough comparison, the Cooler intrinsic damping and diffusive terms are sometimes neglected in the simulation in order to save time. This is because, the diffusive growth studied here occurs in the first 10 to 20 ms, while the Cooler intrinsic damping time is ~ 0.4 s.

Figure 2 shows the simulations when the relative phases are $\Delta\phi_0 = 245°$ and 180°. The other parameters, $f_m = 1400$ Hz, $a = 100°$, $r = 0.11$, $f_{s0} = 667$ Hz, are held fixed. We see that the growth depends very crucially on the relative phase. There is no growth when $\Delta\phi_0 = 180°$; but the growth at 245° is big with large-amplitude oscillations. The Poincaré surfaces of section shown in Fig. 3 tell the reason. For $\Delta\phi_0 = 180°$, the tori near the center are well behaved, therefore the growth if any is limited. However, when $\Delta\phi_0 = 245°$, there is a chaotic region due to overlapping resonances and the bunch grows to the size of the chaotic region.

To understand the significance of the large-amplitude oscillations in the rms beam width σ_t^2 observed in Fig. 2 when $\Delta\phi_0 = 245°$, we show in Fig. 4 the evolution of the phase-space distribution of the bunch during the secondary rf phase modulation for the first 12 consecutive modulation periods. It is evident that the particles

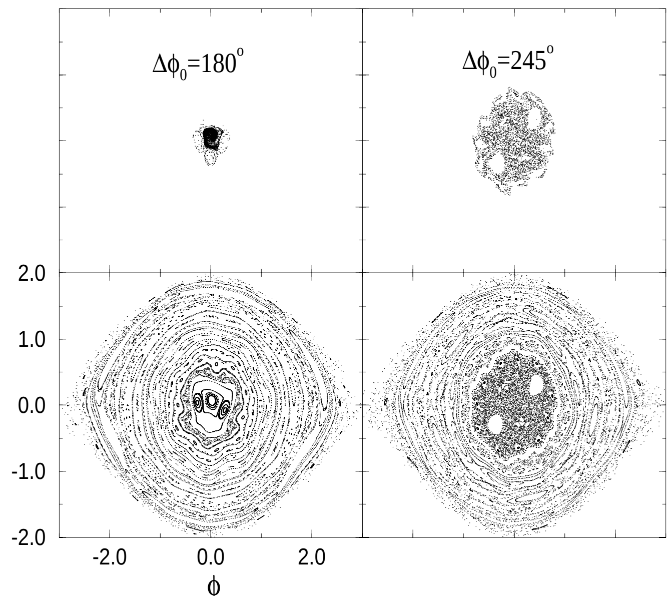

Figure 3: *Poincaré surfaces of section (bottom plots) and the final beam distribution obtained from numerical simulations (top plots) for relative phase differences* $\Delta\phi_0$=180° *(left) and* 245° *(right).*

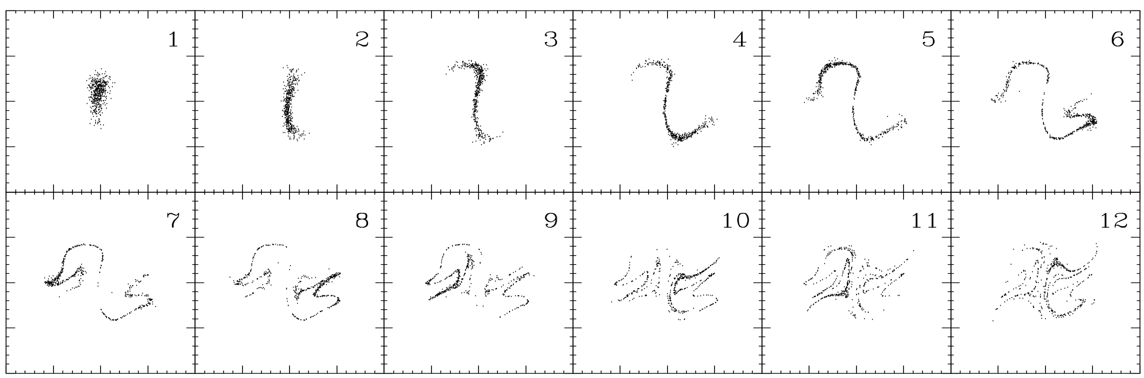

Figure 4: *Phase space evolution for the bunch dilution in Fig. 2 when* $\Delta\phi_0 = 245°$.

follow the separatrices of the 2:1 resonance, which is still dominant although highly broken. The maximum σ_t^2 corresponds to the time that particles diffuse into the maximum of the dominant parametric resonance. As particles gradually fill the chaotic sea of overlapping parametric resonances, the corresponding oscillatory amplitude in σ_t^2 will decrease as well. The final equilibrium bunch length is given by the phase space area of chaotic region.

Figure 5 shows the situation when f_m = 1600 Hz, f_{s0} = 667 Hz, r = 0.20, $a = 140°$, and $\Delta\phi_0 = 280°$. We see that the growth is almost linear initially with smaller amplitude oscillations and then saturated. This can be understood easily by the presence of a large chaotic region in the Poincaré section. The successive modulation-period plots also indicate that the particles follow the dominating 2:1 resonance for much shorter time than the situation in Fig. 4, implying that the resonance has been much more broken.

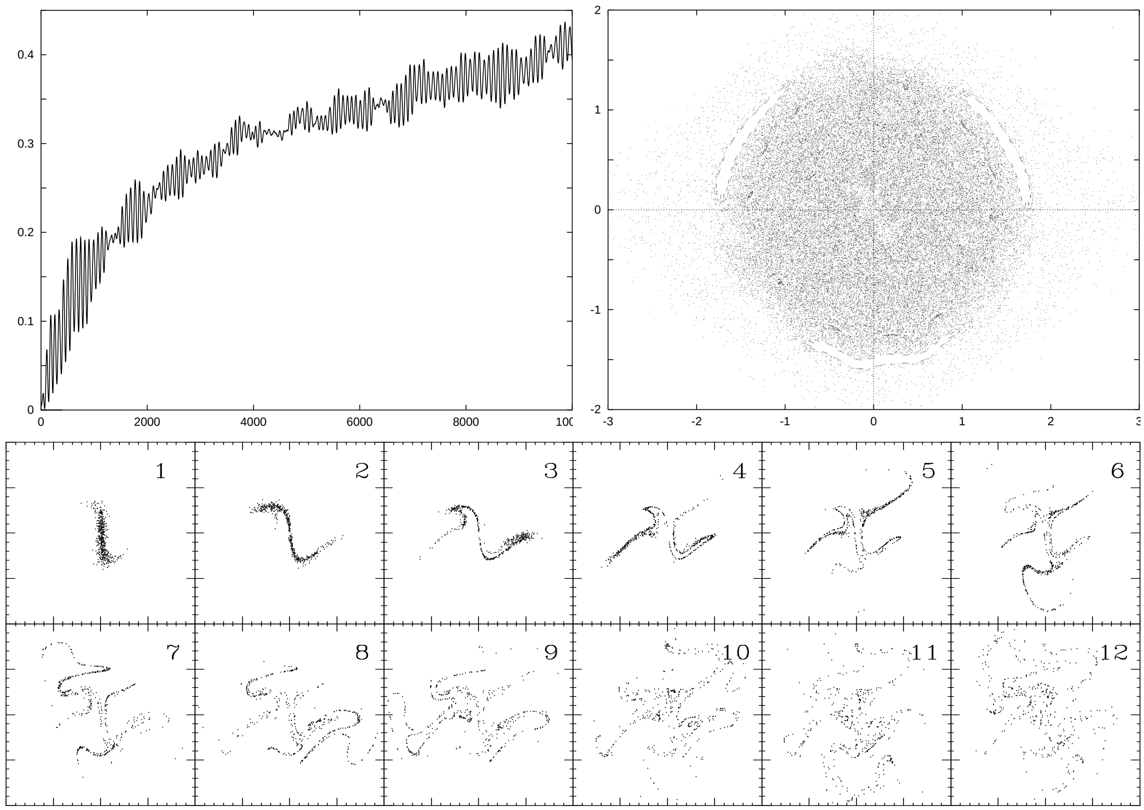

Figure 5: *Simulation results for the situation $f_m = 1600$ Hz, $f_{s0} = 667$ Hz, $r = 0.20$, $a = 140°$, $\Delta\phi_0 = 280°$. Top left plot is growth of σ_ϕ^2 versus turn number.*

Figure 6 shows the situation when $f_m = 2700$ Hz, $f_{s0} = 655$ Hz, $r = 0.233$, $a = 67°$, and $\Delta\phi_0 = 280°$. The initial growth is nonlinear. This is because although a large part of the Poincaré section is chaotic, it is embedded with many islands, which slow the diffusion process. Note also that the total growth is smaller than that in Fig. 5.

4 CONCLUSIONS

Under some conditions when harmonic phase modulation is applied to a double rf system, stochastic region develops. The bunch dilution is a result of diffusion in the stochastic region, and is not driven by a single parametric resonance. How much the beam diffuses or the final bunch size depends solely on how large the KAM-torus-bound stochastic region is. If some dominant parametric resonance is not completely destroyed, the beam will diffuse along the separatrices of this resonance system giving rise to large-amplitude oscillations and fast diffusion. As the particles gradually fill the chaotic sea of overlapping parametric resonances, the oscillation in σ_t^2 will decrease as well, and the diffusion will slow down.

When the central region of phase space is completely chaotic, σ_t^2 will increase linearly according to Einstein's relation. The growth slows down or saturates once

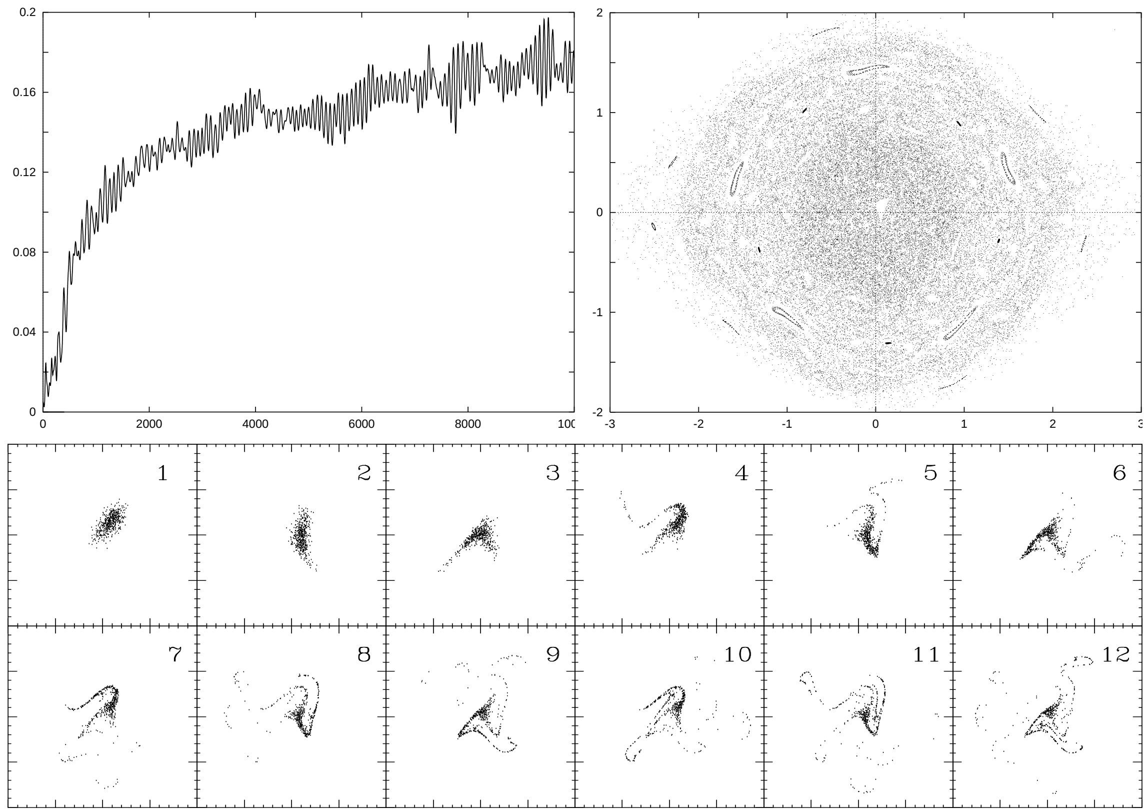

Figure 6: *Simulation results for the situation $f_m = 2700$ Hz, $f_{s0} = 655$ Hz, $r = 0.233$, $a = 67°$, and $\Delta\phi_0 = 280°$. Top left plot is growth of σ_ϕ^2 versus turn number.*

the particles go into an outside less chaotic region.

If the chaotic phase space has partly broken tori embedded inside, it takes time to cross these tori. σ_t^2 will grow nonlinearly, slowly first and then faster.

The area and shape of the stochastic region can hardly be obtained analytically, but can be easily obtained by plotting the Poincaré surface of section. The experimental results agree reasonably well with simulations. The relative phase $\Delta\phi_0$ between the two rf systems turns out to be very crucial in deciding the size of the chaotic region. While $\Delta\phi_0 \approx 0$ or π gives the minimal growth, $\Delta\phi_0 \approx \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ leads to large growth.

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